

On Supersymmetries

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After reviewing electroweak (EW) scale supersymmetry (susy) and split susy, as well as their implications in very high energy cosmic rays, I present a high scale susy model for fermion masses. An $\mathcal{O}(0.1)$ $\nu_e - \nu_\tau$ mixing is expected.

1. Overview of susy

1.1. EW scale susy

Originally, EW scale susy was introduced because of grand unification and naturalness. LEP may imply grand unification of the three gauge couplings at $M_{GUT} \sim 10^{15-16}$ GeV. One generation fermions compose of the $SO(10)$ **16** representation with neutrino masses $m_\nu \sim \frac{m_t^2}{M_{GUT}}$. The Standard Model (SM) Higgs mass $m_h \ll M_{GUT}$, it is unnatural from quantum mechanics! EW scale susy is very required.

Consider very high energy cosmic rays. After hitting the atmosphere they can produce 100 GeV susy particles, $E_{CM} = \sqrt{2m_p E_{CR}}$ can be as large as 100 TeV. However, this is LHC-like with a high background. Specific models may have distinct signals in ultra high energy cosmic rays.

One of the guiding problems in susy studies is the flavor changing neutral current (FCNC) problem. For an example, $\mu \rightarrow e\gamma$ requires certain pattern of slepton masses which result from the susy breaking mechanism.

One solution to the FCNC problem is gauge mediated susy breaking [1]. Susy breaking occurs in a hidden sector, the breaking transfers to the SM sector via gauge interactions, sleptons are degenerate. Taking the susy breaking scale as \sqrt{F} , $m_{slepton} \sim \frac{\alpha}{4\pi} \frac{F}{M}$ with M being messenger masses, $\sqrt{F} \geq 100$ TeV. The gravitino mass is $m_{gravitino} \sim \frac{F}{M_{plank}} \ll$ EW scale. Such

a gravitino can be the dark matter! The next lightest stable susy particle (NLSP) is stau: $\tilde{\tau}_R$ (~ 100 GeV). Its lifetime ($\tilde{\tau} \rightarrow \tau + \text{gravitino}$)

$$\tau_{\tilde{\tau}_R} \sim \frac{16\pi F^2}{m_{\tilde{\tau}_R}^5} \text{ which can be long enough (1 sec)!}$$

Long-lived charged particles can be seen in cosmic rays. In Ref. [2], by taking $\sqrt{F} \simeq 5 \times 10^6 - 5 \times 10^8$ GeV, it was obtained that

$$c\tau_{\tilde{\tau}_R} \sim \left(\frac{\sqrt{F}}{10^7 \text{ GeV}} \right)^4 \left(\frac{100 \text{ GeV}}{m_{\tilde{\tau}_R}} \right)^5 10 \text{ km}. \quad (1)$$

The stau production cross section is smaller than SM processes (muon production) by about 2 orders of magnitude. But, the long lifetime compensates for the small production. Neutrino telescopes can detect these charged NLSPs. The effective detect range is hundred or thousand kilometers in IceCube. Staus are pair produced. Typical signals are two tracks separated by about 100 m. The di-muon background is not significant. A few events are expected per year in IceCube. A similar analysis of a model with a quintessino LSP is presented in the Symposium [3].

1.2. Split susy

However, there maybe no naturalness. The cosmological constant with 10^{120} fine tuning might be just so from the anthropic point of view. Then SM is just the full theory. It is a pity for theorists if there is no susy. Split susy was proposed to give up naturalness while keeping good points of low energy susy.

Split susy keeps all fermions to be around the EW scale, and takes sfermions heavy ≥ 100 TeV

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except for one light Higgs. Then it has GUT and the dark matter! It trivially has no any FCNC problem.

The gluino is very long-lived because it decays via virtual squarks. Its lifetime ($\tilde{g} \rightarrow \text{quark} + \text{anti-quark} + \text{LSP}$)

$$\tau_{\tilde{g}} \sim 3 \times 10^{-2} \text{ sec} \left(\frac{m_{\text{squark}}}{10^9 \text{ GeV}} \right)^4 \left(\frac{1 \text{ TeV}}{m_{\tilde{g}}} \right)^5. \quad (2)$$

In Ref. [4], gluino pair production in ultra high energy cosmic rays is analyzed. Neutrino telescopes can detect down-going pairs. One event is expected in IceCube per year when $m_{\tilde{g}} < 170 \text{ GeV}$.

In Ref. [5], air showers caused by cosmic gluino-contained hadrons are analyzed. The very low inelasticity of \tilde{g} -air interaction guarantees the primary particle retains most of its energy traveling to the ground, while the cascade particles behave as in ordinary air showers. Pierre Auger Observatory can detect such showers if cosmic sources are able to accelerate particles above $5 \times 10^{13} \text{ GeV}$.

2. Susy for fermion masses

If susy is not for stabilizing the EW energy scale, what else is this beautiful mathematical physics used for in reality? We propose susy for flavors [6]! The flavor puzzle lies in the pattern of fermion masses, mixing and CP violation. It is observed that for masses of fermion generations: $3rd \gg 2nd \gg 1st$. We propose a family symmetry: Z_{3L} of the $SU(2)_L$ doublets of leptons and quarks, under which $L_1, Q_1 \rightarrow L_2, Q_2 \rightarrow L_3, Q_3 \rightarrow L_1, Q_1$. It results in $m_\tau \neq 0$, $m_t \neq 0$, $m_b \neq 0$ only. The crucial question is that how does Z_{3L} break? For leptons, we have noted [7] that **sneutrino vacuum expectation value (vev)** $v_i \neq 0$, LLE^c **contributes to charged lepton masses** where E^c stands for $SU(2)_L$ singlet leptons. But, one neutrino mass $m_\nu \sim \frac{(g_2 v_i)^2}{M_{\tilde{Z}}} \sim 100 \text{ MeV}$ with g_2 being the gauge coupling and gaugino masses $M_{\tilde{Z}}$ around the EW scale, it is too large! We will make gauginos enough heavy.

Let us describe the model. In addition to gauge

symmetries and susy, Z_{3L} is assumed. It breaks in soft terms. In writing the Lagrangian, both kinetic terms and the superpotential consist of most general Z_{3L} terms. The canonical kinetic form is got via field redefinition. Finally, the superpotential becomes

$$\mathcal{W} = -y_\tau H_d L_\tau E_\tau^c + L_e L_\mu (\lambda_\tau E_\tau^c + \lambda_\mu E_\mu^c) + \bar{\mu} H_u H_d, \quad (3)$$

where $H_{u,d}$ are the two Higgs doublets, $\bar{\mu}$ a mass parameter and $y_\tau, \lambda_{\tau,\mu}$ couplings. We see that H_d contributes to the tau mass only; sneutrinos in L_e and L_μ to the muon mass, and the electron remains massless.

It is important to note that masslessness of the electron is kept by susy. Generally, family symmetries keep the muon and electron massless. Once the family symmetry is broken, however, both muon and electron get their masses. And there is no reason to expect a hierarchy between the muon mass and the electron mass. In this model, it is the simplicity of the superpotential that makes the electron massless even if sneutrino vevs are non-vanishing. The simplicity comes from susy. The non-vanishing electron mass is therefore due to susy breaking effects.

Soft susy terms breaking Z_{3L} can be written. It has been shown that by fine-tuning, one Higgs scalar doublet with a mass-squared $-m_{EW}^2$ can be obtained. EW symmetry breaking is achieved. The tuning is at the order of m_S^2/m_{EW}^2 where m_S is the susy breaking scale. This light Higgs is a mixture of H_u, H_d and sleptons in Eq. (3). In terms of the latter fields,

$$v_u \neq 0, v_d \neq 0, v_{l_\alpha} \neq 0 \quad (\alpha = e, \mu, \tau). \quad (4)$$

Numerically $v_d \sim 10 \text{ GeV}$ and $v_{l_\alpha} \sim 1 \text{ GeV}$.

Is a large v_{l_α} safe? In addition, huge slepton-Higgs scalar mixing mass-squared soft terms induce large lepton-Higgsino mixing at the loop-level, $m_{\alpha h} = \frac{g_2^2 B_{\mu_\alpha}}{16\pi^2 M_{\tilde{Z}}}$ which is about $10^{-3} m_S$.

The neutralino ($\nu_e \nu_\mu \nu_\tau \tilde{h}_d^0 \tilde{h}_u^0 \tilde{Z}$) mass matrix

is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & m_{eh} & av_{l_e} \\ 0 & 0 & 0 & 0 & m_{\mu h} & av_{l_\mu} \\ 0 & 0 & 0 & 0 & m_{\tau h} & av_{l_\tau} \\ 0 & 0 & 0 & 0 & -\tilde{\mu} & av_d \\ m_{eh} & m_{\mu h} & m_{\tau h} & -\tilde{\mu} & 0 & -av_u \\ av_{l_e} & av_{l_\mu} & av_{l_\tau} & av_d & -av_u & M_{\tilde{Z}} \end{pmatrix} \quad (5)$$

where $a = (\frac{g_2^2 + g_1^2}{2})^{1/2}$, and \tilde{h} stands for higgsinos. Its large eigenvalues are the following

$$\Lambda_1 \simeq M_{\tilde{Z}}, \quad \Lambda_2 \simeq \tilde{\mu}, \quad \Lambda_3 \simeq -\tilde{\mu}. \quad (6)$$

There are three light neutrinos. This is a realization of the see-saw mechanism with heavy higgsinos and gauginos playing the role of right-handed neutrinos. The light Majorana neutrino mass matrix is then

$$\begin{aligned} m^\nu &\simeq -m_{\text{Dirac}} M_R^{-1} m_{\text{Dirac}}^T, \\ &= -\frac{a^2}{M_{\tilde{Z}}} \begin{pmatrix} v_{l_e} v_{l_e} & v_{l_e} v_{l_\mu} & v_{l_e} v_{l_\tau} \\ v_{l_\mu} v_{l_e} & v_{l_\mu} v_{l_\mu} & v_{l_\mu} v_{l_\tau} \\ v_{l_\tau} v_{l_e} & v_{l_\tau} v_{l_\mu} & v_{l_\tau} v_{l_\tau} \end{pmatrix}. \end{aligned} \quad (7)$$

It is a democratic matrix for neutrinos, naturally large neutrino mixing are expected. The nonvanishing mass is $m_{\nu_3} = \frac{a^2}{M_{\tilde{Z}}} v_{l_\alpha} v_{l_\alpha} \sim 10^{-1} - 10^{-2}$ eV when $M_{\tilde{Z}} \sim 10^{11} - 10^{12}$ GeV.

The electron mass is due to soft susy breaking terms, $\delta M_{\alpha\beta}^l \simeq \frac{\alpha}{\pi} \frac{y_\tau \tilde{m}_S v_d}{m_S}$, with \tilde{m}_S being trilinear soft masses. Taking $\tilde{m}_S/m_S \simeq 0.1$, $\delta M_{\alpha\beta}^l \sim \mathcal{O}(\text{MeV})$.

Neutrino oscillation should be analyzed. One SM singlet superfield N is needed,

$$\mathcal{W} \supset \kappa_\tau H_u L_\tau \bar{N} + \tilde{M} \bar{N} \bar{N} + \kappa_d H_u H_d \bar{N} + \tilde{\kappa}_3 \bar{N}^3, \quad (8)$$

where κ 's are couplings and \tilde{M} the mass of N . The full neutrino mass matrix is

$$\mathcal{M}^\nu = -\frac{a^2}{M_{\tilde{Z}}} \begin{pmatrix} v_{l_e} v_{l_e} & v_{l_e} v_{l_\mu} & v_{l_e} v_{l_\tau} \\ v_{l_\mu} v_{l_e} & v_{l_\mu} v_{l_\mu} & v_{l_\mu} v_{l_\tau} \\ v_{l_\tau} v_{l_e} & v_{l_\tau} v_{l_\mu} & v_{l_\tau} v_{l_\tau} + x \end{pmatrix} \quad (9)$$

with x being $\frac{M_{\tilde{Z}}}{\tilde{M}} \left(\frac{\kappa_\tau v_u}{a} \right)^2$. Its eigen values are

$$\begin{aligned} m_{\nu_3} &\simeq \frac{a^2}{M_{\tilde{Z}}} v_{l_\tau}^2 + \frac{(\kappa_\tau v_u)^2}{\tilde{M}}, \\ m_{\nu_2} &\simeq \frac{a^2}{M_{\tilde{Z}}} (v_{l_e}^2 + v_{l_\mu}^2) \frac{x}{x + v_{l_\tau}^2}, \\ m_{\nu_1} &= 0. \end{aligned} \quad (10)$$

Solar neutrino problem requires that $m_{\nu_2} \simeq (10^{-2} - 10^{-3})$ eV which is achieved when $M_{\tilde{Z}} \sim 10^{13}$ GeV. Atmospheric neutrino problem requires a certain cancellation between the terms $\frac{a^2}{M_{\tilde{Z}}} v_{l_\tau}^2$ and $\frac{(\kappa_\tau v_u)^2}{\tilde{M}}$, in order to make $m_{\nu_3} \sim 10^{-1} - 10^{-2}$. The lepton mixing are

$$|V_{e2}| = \frac{v_{l_\mu}^2 - v_{l_e}^2}{v_{l_e}^2 + v_{l_\mu}^2} \simeq \mathcal{O}(1). \quad (11)$$

$$|V_{\mu 3}| \simeq \frac{|\lambda_\tau| \sqrt{v_{l_e}^2 + v_{l_\mu}^2}}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}}. \quad (12)$$

The maximal mixing can be achieved. The $\nu_e - \nu_\tau$ mixing is

$$|V_{e3}| \simeq \frac{v_{l_\mu}^2 - v_{l_e}^2}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2} v_\tau}. \quad (13)$$

It is ~ 0.1 if $\sqrt{v_{l_e}^2 + v_{l_\mu}^2}/v_\tau \sim 0.1$.

Quark masses also have three origins: *Higgs vevs*, *sneutrino vevs*, *soft trilinear Z_{3L} violating terms*. However, the role of the sneutrino vevs and soft trilinear terms are switched. Sneutrino vevs contribute to the first generation quark mass, and soft trilinear Z_{3L} violating terms to charm and strange quark masses. The hierarchy between the second and first generation is not automatic. A special structure of soft breaking terms of squarks is needed. A good point is that $m_u < m_d$ can be understood.

Higgs mass $\simeq 145 \pm 7$ GeV. This was obtained in Ref. [8], which considered a high scale susy scenario in a different physics content. One specific point is that we now have $\tan \beta \simeq m_t/m_b$.

Coming back to cosmic ray physics, our model has little to say compared to SM. One special

point is that we predict a relatively large θ_{13} . This has certain cosmic ray physics implication [9].

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